THE EFFECT OF EMBEDMENT ON FOOTING VIBRATIONS

by

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Synopsis

The forced vibration of embedded footings is investigated both theoretically and experimentally. The approximate analytical solution, based on the side reactions computed by Baranov (2), facilitates the analysis of all vibration modes with any dynamic stress distribution in the foundation base and with various assumptions concerning the back fill. Field experiments are described with concrete footings featuring square and rectangular bases. Vertical and coupled horizontal motions are studied with embedment into both undisturbed soil and back fill. It appears that the main factors are: direction of excitation, depth of embedment and stress distribution in the base and also the density of the back fill and its bond with the footing.

Introduction

Most studies of footing vibrations concern bodies attached to the surface of the soil or, in the theory, to the surface of the elastic half-space. However, the experiments indicate (5), (6), (9), (10) that the footing response can be highly affected by the embedment. It has been recognized that the embedment reduces the resonant amplitudes and increases the resonant frequencies and that these effects are more marked with the horizontal excitation. Quantitatively, these observations remain rather vague.

There are several theoretical solutions based on the elastic half-space theory. Tajimi (14) solved the vibration of a body embedded in an elastic stratum overlying the bedrock. Baranov (2) considered the reactions in the base from a half-space and derived side reactions from an independent horizontal layer overlying the half-space. Lysmer and Kuhlemeyer (8) solved the vertical vibration using the finite element method. The solutions by Baranov and Lysmer and Kuhlemeyer assumed that the soil around the footing is the same as beneath its base. All solutions assumed, furthermore, that there is a perfect bond between the footing sides and the soil. These two assumptions are not satisfied if the footing is poured into forms and then surrounded by a back fill as is often the case. Experiments indicate (10) that in such cases the effect of embedment can be considerably reduced and the theory may predict much lower resonant amplitudes and higher resonant frequencies than are probable to appear in reality.

As for resonant amplitudes there is also another possible reason for the theory not to yield conservative results. Sung's approximate solution

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(13) indicates that the resonant amplitudes markedly vary with the assumed stress distribution in the base and that the parabolic distribution yields the maximum response. Experimental amplitudes from some field tests (10) appear much closer to the theoretical amplitudes computed under the assumption of the parabolic stress distribution, than to the results of the exact solution to the contact problem (1) or to the approximate solution with a rigid base stress distribution (13). Chae's laboratory tests (4) suggest that the dynamic stress distribution varies with frequency. Furthermore, it can be expected that the stress distribution also depends upon the kind of soil, size of the footing and the level of stress. With respect to these factors, it seems desirable that the theory be flexible enough to comply with the experimental findings.

Here, some theoretical results are outlined and field experiments with concrete test footings presented. The main goal is to accumulate information about the major factors dominating the dynamic response of embedded footings.

Theoretical Solution

From the point of view of versatility, an approximate analytical approach seems suitable in which the reaction in the footing base is derived from the half-space and the soil reactions acting on the sides are taken as the forces produced by an independent overlying stratum. This approach was applied by Baranov (2) who derived relations for the dynamic reactions of a layer having a unit thickness and attached to a cylindrical footing with radius r_o. The relation between the complex vertical motion $w(t) = w\ exp\ (i\omega t)$ and the vertical reaction of the unit layer $s_w(t)$ is

$$s_w(t) = G_s(S_{w1} + iS_{w2})\ w(t) \tag{1}$$

in which

$$S_{w1} = 2\pi a_o \frac{J_1J_o + Y_1Y_o}{J_o^2 + Y_o^2}, \quad S_{w2} = \frac{4}{J_o^2 + Y_o^2} \tag{2}$$

Horizontal translation of the footing $u(t) = u\ exp\ (i\omega t)$ produces a horizontal reaction of the unit layer

$$s_u(t) = G_s(S_{u1} + iS_{u2})\ u(t) \tag{3}$$

in which with $\nu = 0.5$

$$S_{u1} = \pi(1 + 2\frac{J_oJ_2 + Y_oY_2}{J_o^2 + Y_o^2})\ a_o^2, \quad S_{u2} = \frac{8}{J_o^2 + Y_o^2} \tag{4}$$

Finally, rotation about a horizontal axis $\psi(t) = \psi\ exp\ (i\omega t)$ produces a moment

$$s_\psi(t) = G_s r_o^2\ (S_{\psi 1} + iS_{\psi 2})\ \psi(t) \tag{5}$$

where

$$S_{\psi 1} = \pi(1 - a_o\frac{J_oJ_1 + Y_oY_1}{J_1^2 + Y_1^2}), \quad S_{\psi 2} = \frac{2}{J_1^2 + Y_1^2} \tag{6}$$

In the above relations, $J_o(a_o)$, $J_1(a_o)$, $J_2(a_o)$ = Bessel functions of the first kind of order zero, one and two respectively and $Y_o(a_o)$, $Y_1(a_o)$, $Y_2(a_o)$ = Bessel functions of the second kind of order zero, one and two. The argument is the dimensionless frequency $a_o = r_o \omega \sqrt{\rho/G_s}$, G_s = shear modulus and ρ = mass density of the layer.

The soil reactions in the footing base can be considered approximately equal to the elastic half-space reactions readily available for all vibration modes in the literature (3), (13).

The vertical reaction with a pure vertical motion $w(t)$ can be written as:

$$R_z(t) = - Gr_o \left(\frac{1}{f_{w1} + if_{w2}}\right) w(t) = Gr_o(C_{w1} + iC_{w2})\, w(t) \qquad (7)$$

The horizontal reaction with a pure horizontal vibration $u(t)$ is similarly

$$R_x(t)= -Gr_o\left(\frac{1}{f_{u1} + if_{u2}}\right) u(t) = Gr_o\, (C_{u1} + iC_{u2})\, u(t) \qquad (8)$$

With pure rocking, $\psi(t)$, the reactive moment is analogous

$$R_\psi(t) = - Gr_o^3 \left(\frac{1}{f_{\psi 1} + if_{\psi 2}}\right) \psi(t) = Gr_o^{\,3}\, (C_{\psi 1} + iC_{\psi 2})\, \psi(t) \qquad (9)$$

In Equations (7), (8) and (9), G = shear modulus of the half-space (soil beneath the base) and

$$C_{w1} = \frac{-f_{w1}}{f_{w1}^2 + f_{w2}^2}; \qquad C_{w2} = \frac{f_{w2}}{f_{w1}^2 + f_{w2}^2} \qquad (10)$$

$$C_{u1} = \frac{-f_{u1}}{f_{u1}^2 + f_{u2}^2}; \qquad C_{u2} = \frac{f_{u2}}{f_{u1}^2 + f_{u2}^2} \qquad (11)$$

$$C_{\psi 1} = \frac{-f_{\psi 1}}{f_{\psi 1}^2 + f_{\psi 2}^2}; \qquad C_{\psi 2} = \frac{f_{\psi 2}}{f_{\psi 1}^2 + f_{\psi 2}^2} \qquad (12)$$

Functions f are taken with the same signs as in (3), (13). Then, the equations of motion for the three uncoupled vibration modes are:

$$m\ddot{w} + Gr_o\{C_{w1} + iC_{w2} + \frac{G_s}{G}\,\delta(S_{w1} + iS_{w2})\}\, w(t) = P_o\, exp\, (i\omega t) \qquad (13)$$

$$m\ddot{u} + Gr_o\{C_{u1} + iC_{u2} + \frac{G_s}{G}\,\delta(S_{u1} + iS_{u2})\}\, u(t) = Q_o\, exp\, (i\omega t) \qquad (14)$$

$$I_\psi \ddot{\psi} + Gr_o^3 \{C_{\psi 1} + iC_{\psi 2} + \frac{G_s}{G} [\delta(S_{\psi 1} + iS_{\psi 2}) + \frac{\delta^3}{3}(S_{u1} + iS_{u2})]\}\, \psi(t)$$

$$= T_o \exp(i\omega t) \qquad (15)$$

in which m = mass of footing, I_ψ= mass moment of inertia about the centre of the base, $\delta = l/r_o$ = relative embedment, P_o = amplitude of the vertical exciting force, Q_o = amplitude of the horizontal exciting force and T_o = amplitude of the exciting moment about the centre of the base. Functions $S(a_o)$ and $C(a_o)$ are given by Equations (2), (4), (6) and (10), (11), (12).

From Equations (13), (14) and (15), the steady forced oscillations can be solved in a straight-forward manner.

Baranov's approach can be extended to involve various stress distributions in the footing base and various assumptions concerning the back fill. Both of these factors can be substantial. Finally, the results of the solution can be presented in dimensionless form suitable for applications.

Considering only the moment from the horizontal force $Q(t)$ and assuming a frequency variable excitation

$$P_o = Q_o = m_e e\omega^2 \text{ and } T_o = m_e e\omega^2 z_b \qquad (16)$$

in which m_e, e = rotating mass and its excentricity and z_b = the height of $Q(t)$ above the base. Denote the real amplitudes of motion w_o, u_o and ψ_o. Then the dimensionless real amplitudes can be introduced:

$$A_z = \frac{m}{m_e e} w_o\,,\; A_x = \frac{m}{m_e e} u_o,\; A_\psi = \frac{I_\psi}{m_e e z_b} \psi_o \qquad (17)$$

With this notation the dimensionless amplitudes of vertical, horizontal and rocking vibrations of embedded footings are, from Equations (13) to (16):

$$A_z = m\omega^2 \{(k_z - m\omega^2)^2 + (c_z\omega)^2\}^{-\frac{1}{2}}$$

$$A_x = m\omega^2 \{(k_x - m\omega^2)^2 + (c_x\omega)^2\}^{-\frac{1}{2}} \qquad (18)$$

$$A_\psi = I_\psi\omega^2 \{(k_\psi - I_\psi\omega^2)^2 + (c_\psi\omega)^2\}^{-\frac{1}{2}}$$

in which

$$k_z = Gr_o\{C_{w1} + \frac{G_s}{G} \delta S_{w1}\}$$

$$k_x = Gr_o \{C_{u1} + \frac{G_s}{G} \delta S_{u1}\}$$

$$k_\psi = Gr_o^3 \{C_{\psi 1} + \frac{G_s}{G}\delta (S_{\psi 1} + \frac{1}{3}\delta^2 S_{u1})\} \qquad (19)$$

$$c_z = \frac{Gr_o}{\omega} \{C_{w2} + \frac{G_s}{G} \delta S_{w2}\}$$

$$c_x = \frac{Gr_o}{\omega} \{C_{u2} + \frac{G_s}{G} \delta S_{u2}\}$$

$$c_\psi = \frac{Gr_o^{\,3}}{\omega} \{C_{\psi 2} + \frac{G_s}{G} \delta (S_{\psi 2} + \frac{1}{3} \delta^2 S_{u2}\}$$

Equations (18) for A_z, A_x, and A_ψ are formally equal to the dimensionless amplitudes of single degree of freedom systems. Thus k_z, k_x and k_ψ are equivalent spring constant and c_z, c_x and c_ψ are equivalent damping constants for an embedded footing. Therefore, the undamped natural frequencies of embedded footings are:

$$\omega_{oz} = \sqrt{\frac{Gr_o}{m} \{C_{w1} + \frac{G_s}{G} \delta S_{w1}\}}$$

$$\omega_{ox} = \sqrt{\frac{Gr_o}{m} \{C_{u1} + \frac{G_s}{G} \delta S_{u1}\}} \qquad (20)$$

$$\omega_{o\psi} = \sqrt{\frac{Gr_o^{\,3}}{I_\psi} \{C_{\psi 1} + \frac{G_s}{G} \delta (S_{\psi 1} + \frac{1}{3} \delta^2 S_{u1})\}}$$

The natural undamped frequencies ω_o must be determined from Equation (20) by trial and error approach since they appear in C and S too. (The undamped natural frequency is not equal to the frequency at maximum amplitude which is called resonant frequency herein.)

Examples of theoretical response curves at various embedments are presented in Figs. 1 to 4. The vertical (Fig. 1, 2) and horizontal (Fig. 3) vibrations were computed from Equation (18) with $\nu = 0.5$ while for rocking vibrations (Fig. 4) $\nu = 0.0$ was used.

The dependence of the response curves upon Poisson's ratio is reduced when modified mass ratios B are used. {These were introduced by Lysmer and Hall (12)}. Both mass ratios b_o and B are given in the figures.

The theoretical response curves shown in Fig. 1 were computed with Bycroft's (3) displacement functions $f_{1,2}$. The curves shown in full lines represent a situation in which the soil properties are the same beneath the footing and around it as is the case with footings embedded in undisturbed soil. These response curves are in reasonable agreement with those by Lysmer and Kuhlemeyer (8) obtained by finite element approach.

Curves shown in the dashed lines are computed for a footing surrounded by a back fill whose density $\rho_s = 0.75\rho$. {It is assumed that $G_s/G \simeq (\rho_s/\rho)^3$}. It can be seen that the effect of embedment is reduced in the case of back fill.

The reduction of resonant amplitudes and the increase in resonant frequencies qualitatively agrees with experimental observations. However, the theoretical resonant amplitudes are roughly one half of those observed in previous experiments (10). This discrepancy can be considerably reduced if a parabolic stress distribution is assumed in the footing base. The theoretical response curves computed under this assumption with Sung's (13) values of frequency functions f_1, f_2 are plotted in Fig.2. It can be seen that response amplitudes are much larger in this case. The parabolic stress distribution seems rather speculative. Nevertheless, there are indications that it may be sometimes acceptable {Chae (4)}. Anyway, the parabolic stress distribtuion combined with the assumption of a back fill yields the upper bound for theoretical amplitudes of vibrations.

The theoretical response curves for pure horizontal translation are plotted in Fig. 3. The theoretical response curves for pure rocking vibration (rotation ψ) are given in Fig. 4. As can be seen, the effect of embedment is by far most pronounced with pure rocking vibrations.

It might be noted that pure horizontal translation and pure rocking imply the existance of constraints eliminating other components of motion. These constraints, however, do not exist in reality. Therefore the horizontal excitation always produces a coupled motion composed of horizontal translations and rotation; with an assymmetric footing a vertical component is also present. Thus, Fig. 3 approximately illustrates the horizontal vibration of a very flat footing and Fig. 4 the rotation of a very tall footing. The theory outlined can be used to solve the coupled motion too. This is presented elsewhere. The theory is approximate in that the compatibility condition between the half-space and the overlying layer is neglected. However, the results seem reasonable.

The experimental results available so far indicate that the real effect of embedment may also depend on some further factors not included in the theory. There may also be some difference in the magnitude of the theoretical and experimental amplitudes as mentioned above. To shed some light on these questions, field experiments described herein were carried out.

Experiments With Embedded Footings

Two concrete blocks with base areas of 5 square feet were used in the field tests. One block had a square base, the other one a rectangular base with side ratio 2/1. The blocks were poured directly into excavations. The initial embedment was 36 inches. The embedment depth was changed by removing the soil in several steps. Then the soil was step-wise back filled and tamped to two different densities. The subsoil was composed of about 5 feet of brown silty clay underlain by a glacial till of considerable thickness.

The test equipment consisted of:

(i) A Lazan Mechanical Oscillator producing a frequency dependent exciting force with a maximum eccentric moment of 18 lb-in in either vertical or horizontal directions.

(ii) A 220-Volt three-phase motor fitted with a Kopp Variator, providing stepless speed variation from 300 to 3600 revolutions per minute, and connected with the oscillator by a flexible shaft.

(iii) A dual beam storage oscilloscope.

(iv) A Brush two channel recorder.

(v) Two IRD electrodynamical vibration pick-ups which measured absolute quantities.

(vi) A portable self-powered IRD Vibration Meter (Model 306).

(vii) A "Strobotac" Stroboscope for accurate measurement of speed.

The experimental equipment was acquired in connection with a broader research program at The University of Western Ontario. The field tests were carried out by the junior author.

The shear modulus and the Poisson's ratio of the subsoil were obtained in the field from wave length and wave velocity measurements. From these tests and laboratory testing of the soil samples, the following properties were obtained for the dynamic calculations:

Mean bulk density of undisturbed soil $= 103.0\ lb/ft^3$
Mean bulk density of heavily tamped fill $= 98.0\ lb/ft^3$
Mean bulk density of lightly tamped fill $= 79.0\ lb/ft^3$
Mean moisture content of soil $= 16\%$; *void ratio of undisturbed soil* $= 0.9$
Shear modulus of undisturbed soil $= 6.6 \times 10^5 lb/ft^2$
Poisson's ratio of undisturbed soil $= 0.38$

The response curves of steady-state vibrations were measured with vertical or horizontal excitations at various embedment depths. With horizontal excitation, the response measured on the surface was recalculated to yield the horizontal translation and the rotation at the centre of gravity of the footing. Examples of the experimental response curves are given in Figs. 5 to 10. Also given in these figures are the excitation direction and intensity, the depth of embedment and the relative density of the back fill (eg_o = eccentric moment in lb-in).

As can be seen from Figs. 5 and 6, the response curves feature a strong nonlinearity with both surface and embedded footings. The non-linearity exhibits the familiar decrease of resonant frequencies with increasing excitation (10) and complicates the analysis of experiments (11) as well as their comparison with the theory.

Fig. 7 shows the differences observed between the vertical vibrations of square and rectangular footings having the same base areas and masses. (The excitation was also the same in both cases).

Fig. 8 illustrates the horizontal component of coupled vibrations of the square footing embedded into undisturbed soil and excited by a horizontal force. Fig. 9 shows the same kind of motion of the same footing embedded into the back fill.

In Figure 10 the horizontal components of coupled vibrations are presented for the rectangular footing embedded in undisturbed soil. The figure illustrates the difference in response due to the direction of the exciting force.

Comparison of Theory with Experiments

The theoretical response qualitatively agrees with the experimental observations in the following respects: the resonant amplitudes decrease and the resonant frequencies increase with increase in depth of embedment and density of the back fill. The quantitative comparison is affected by the assumptions accepted for the surface footing and by the possible layering of the subsoil. This difficulty can be reduced by considering the relative changes in resonant frequencies and amplitudes due to embedment. Such comparison can be most easily done with vertical vibration of symmetric footings. In Fig. 11, the relative reduction of vertical resonant amplitudes due to embedment in undisturbed soil is plotted against the relative embedment depth l/r_o. The theory is compared with the field tests and other available data. This can be done directly because the theoretical resonant amplitude ratio appears nearly independent of mass parameter with this kind of motion. The agreement between the theory and experiments appears reasonable.

In the case of the back fill the agreement is not as good, (Fig. 12), in particular with lower embedments. Most experiments indicate smaller amplitude reduction and much smaller increase in resonant frequency than the theory predicts. The reason for these discrepancies may be that the theory assumes a perfect bond between the footing sides and the surrounding back fill. It seems advisable for practical purposes to intuitively reduce the theoretical effect of embedment as suggested in Fig. 12. In the same figure the theoretical curve for undisturbed soil is also plotted (η = 1.00). It can be seen that theories assuming $\eta = \rho_s/\rho = 1$ can considerably over-estimate the effect of embedment into the back fill.

The comparison of the theory with experiments in the case of coupled horizontal vibration and rocking can be seen from Figs. 13 to 16. Fig. 15 indicates that the theory is better able to predict the resonant amplitudes for embedded footings than for surface footings. This is most likely due to the neglect of internal friction in the soil and to very small geometric damping of the surface footings in the rocking vibration mode. Further details are presented elsewhere.

Conclusions

The approximate analytical solution of an embedded footing compares favourably with the finite element solution for vertical vibration and can be used to solve the other modes of vibrations too.

The theory and experiments agree qualitatively in the decrease in resonant amplitudes and increase in resonant frequencies with embedment depth and the density of the back fill.

The quantitative agreement is reasonable for footings embedded into the undisturbed soil, in particular with coupled horizontal vibration. With the back fill, the theory appears to over-estimate the relative effect of embedment. This may be attributed to the imperfect bond between the footing sides and the surrounding back fill. (A perfect bond is assumed in the theory).

The greatest influence of embedment can be expected with vibration modes that approach pure rocking.

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Bibliography

1. Awojobi, A.O., Grootenhuis, P., "Vibrations of Rigid Bodies on Semi-Infinite Elastic Media", Proceedings of the Royal Society of London, Series A, Vol. 287, 1965, pp. 27-63.

2. Baranov, V.A., "On the Claculation of Excited Vibrations of an Embedded Foundation", (in Russian), Voprosy Dynamiki i Prochnocti, No. 14, Polytechnical Institute of Riga, 1967, pp. 195-209.

3. Bycroft, G.N., "Forced Vibrations of a Rigid Circular Plate on a Semi-Infinite Elastic Half Space and on an Elastic Stratum", Philosphical Transactions of the Royal Society, London, Series A, Vol. 248, No. 948, 1956, pp. 327-368.

4. Chae, Y.S., "Vibration of Non-Circular Foundations", Jnl. of soil Mechanics and Foundations Division, Proc. ASCE, November, 1969, pp. 1411-1430.

5. Chae, Y.S., "Dynamic Behaviour of Embedded Foundation-Soil Systems", Highway Research Record, No. 323, 1971, pp. 49-59.

6. Fry, Z.B., "Development and Evaluation of Soil Bearing Capacity, Foundation of Structures", WES Tech. Report No. 3-632, Report 1, 1963.

7. Kaldjian, M.J., Discussion of "Design Procedures for Dynamically Loaded Foundations" by R.V. Whitman and F.E. Richard Jr., Jnl. of soil Mechanics and Foundations Div., Proc. ASCE, Vol. 95, No. SM1, January, 1969, pp. 364-366.

8. Lysmer, J. and Kuhlemeyer, R.L., "Finite Dynamic Model for Infinite Media", Jnl. of Engineering Mechanics Division, Proc. ASCE, Vol. 95, No. EM4, August, 1969, pp. 859-877; Closure to Discussions, February, 1971, pp. 129-131.

9. Novak, M., "The Vibration of Massive Foundations on Soil", Int. Association for Bridge and Structural Engineers, Publications No. 20, 1960, pp. 263-281.

10. Novak, M., "Prediction of Footing Vibrations", Jnl. of Soil Mechanics and Foundations Div., Proc. ASCE, Vol. 96, No. SM3, May, 1970, pp. 837-861.

11. Novak, M., "Data Reduction from Nonlinear Response Curves", Jnl. of Engineering Mechanics Division, Proc. ASCE, EM1, August, 1971.

12. Richart, F.E., Hall, J.R., Woods, R.D., "Vibrations of Soils and Foundations", Prentice-Hall Inc., 1970.

13. Sung, T.Y., "Vibrations in Semi-Infinite Solids due to Periodic Surface Loading", Am. Soc. for Testing and Materials Special Tech. Publ. No. 156, Symposium on Dynamic Testing of Soils, 1953, pp. 35-64.

14. Tajimi, H., "Dynamic Analysis of a Structure Embedded in an Elastic Stratum", Proc. 4th World Conference on Earthquake Engineering, Santiago, Chile, 1969, Session A-6, pp. 53-69.

GLOSSARY OF TERMS

A_x = dimensionless amplitude of horizontal vibration

A_z = dimensionless amplitude of vertical vibration

A_ψ = dimensionless amplitude of rocking vibration

a_o = dimensionless frequency = $r_o\omega\sqrt{\rho/G}$

B_x = modified mass ratio for horizontal vibration = $b_o(7-8\nu)/32(1-\nu)$

B_z = modified mass ratio for vertical vibration = $b_o(1-\nu)/4$

B_ψ = modified mass ratio for rocking vibration = $3(1-\nu)I_\psi/8\rho r_o^5$

b_o = mass ratio for vertical and horizontal vibration = $m/\rho r_o^3$

$C_{j1,2}$ = functions of a_o

c_x = equivalent damping coefficients for horizontal vibration

c_z = equivalent damping coefficients for vertical vibration

c_ψ = equivalent damping coefficients for rocking vibration

e = eccentricity of rotating mass

$f_{j1,2}$ = functions of a_o

G = shear modulus of elastic half-space (soil beneath the footing base)

G_s = shear modulus of elastic layer (back fill)

g = acceleration of gravity

g_o = weight of rotating mass = $m_e g$

i = $\sqrt{-1}$

I_ψ = mass moment of inertia about centre of base

k_x = equivalent spring constant for horizontal vibration

k_z = equivalent spring constant for vertical vibration

k_ψ = equivalent spring constant for rocking vibration

l = depth of embedment

m = mass of footing

m_e = rotating (unbalanced) mass

P_o = amplitude of excitation

$P(t)$ = vertical exciting force

Q_o = amplitude of horizontal excitation

$Q(t)$ = horizontal exciting force

$R_x(t)$ = horizontal half-space reaction

$R_z(t)$ = vertical half-space reaction

$R_\psi(t)$ = half-space reactive moment

r_o = radius of footing, equivalent radius of footing

$S_{j1,2}$ = Baranov's frequency functions for layer

$s_j(t)$ = side reaction of layer having unit thickness

T_o = amplitude of exciting moment

t = time

u = horizontal displacement = $u(t)$

u_o = real amplitude of horizontal displacement

w = vertical displacement = $w(t)$

w_o = real amplitude of vertical displacement

z_b = height of $Q(t)$ above footing base

z_c = height of centre of gravity above footing base

z_e = height of $Q(t)$ above centre of gravity

δ = relative embedment = l/r_o

η = density ratio = ρ_s/ρ

ν = Poisson's ratio

ω = excitation frequency

ω_o = natural undamped frequency

ψ = angular (rocking) displacement = $\psi(t)$

ψ_o = real amplitude of angular displacement

ρ = mass density of elastic half-space

ρ_s = mass density of elastic layer

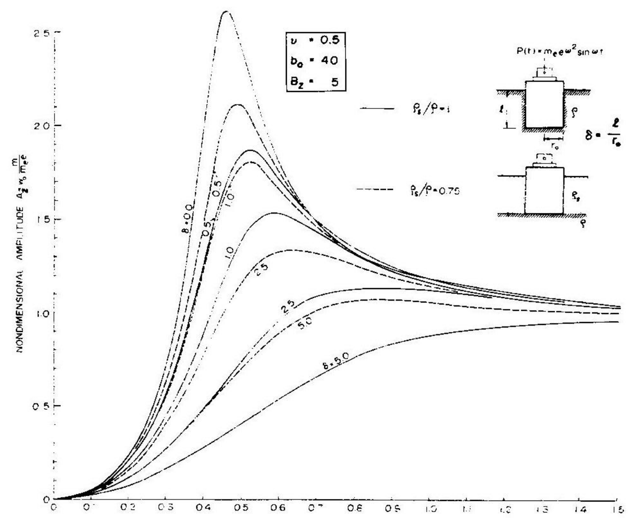

FIGURE 1 THEORETICAL RESPONSE CURVES OF VERTICAL VIBRATIONS OF FOOTINGS EMBEDDED IN UNDISTURBED SOIL (FULL LINES) AND BACK FILL (DASHED LINES). (RIGID BASE STRESS DISTRIBUTION).

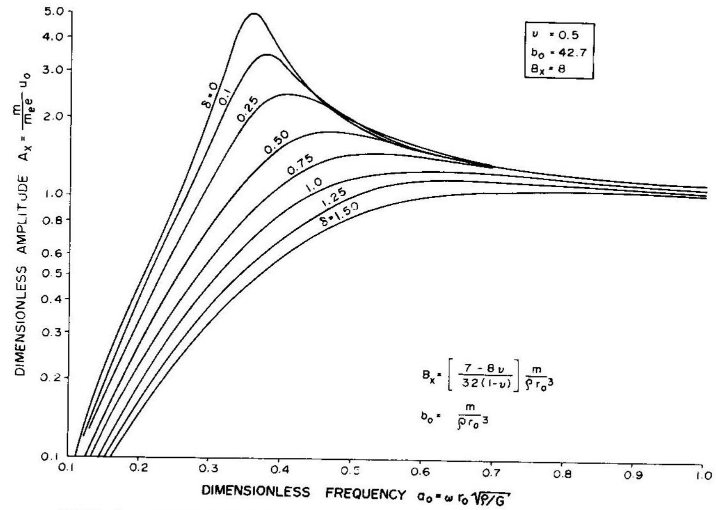

FIGURE 3 THEORETICAL RESPONSE CURVES OF PURE HORIZONTAL TRANSLATION OF FOOTINGS EMBEDDED IN UNDISTURBED SOIL (RELATIVE EMBEDMENT $\delta = \ell/r_o$).

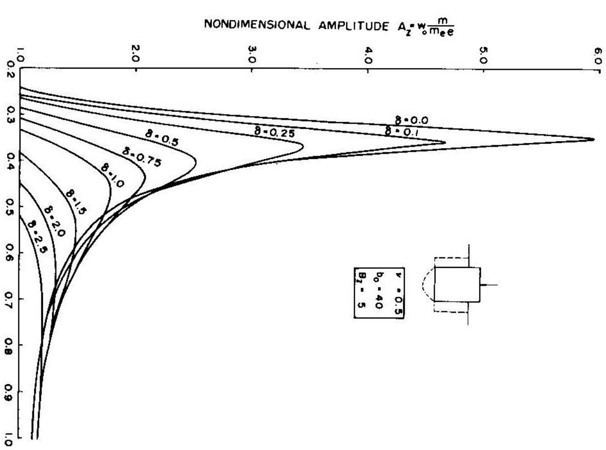

FIGURE 2 THEORETICAL RESPONSE CURVES OF VERTICAL VIBRATIONS OF FOOTINGS EMBEDDED IN UNDISTURBED SOIL. (PARABOLIC STRESS DISTRIBUTION)

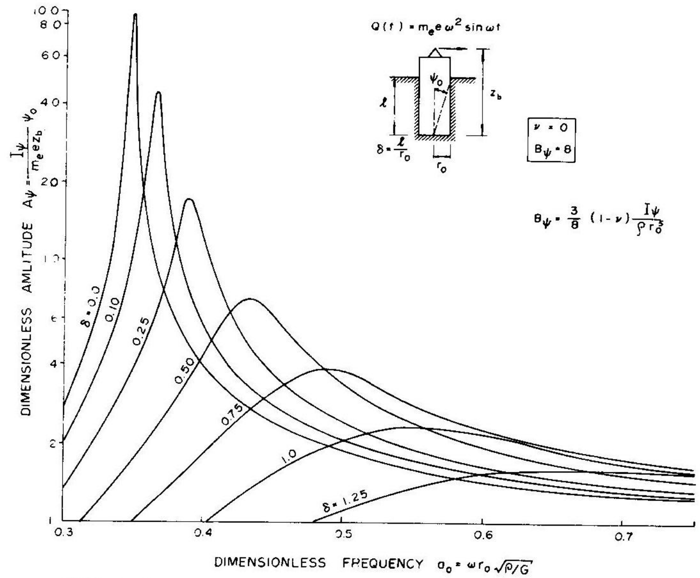

FIGURE 4 THEORETICAL RESPONSE CURVES OF PURE ROCKING VIBRATIONS OF FOOTINGS EMBEDDED IN UNDISTURBED SOIL

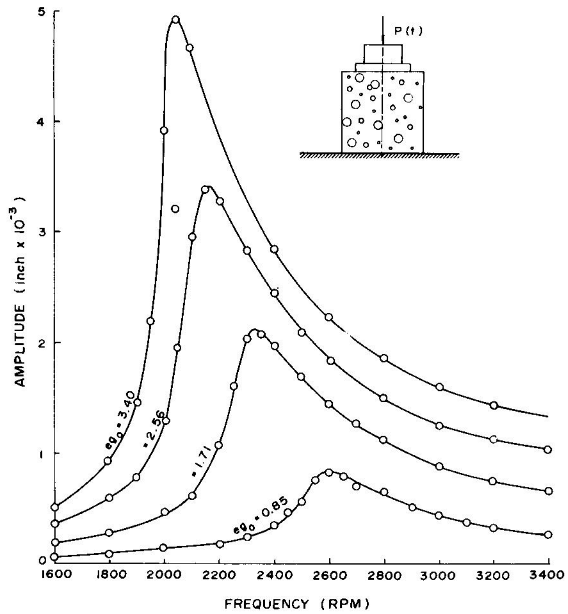

FIGURE 5 VERTICAL VIBRATION OF SURFACE FOOTING WITH SQUARE BASE AT VARIOUS ECCENTRIC MOMENTS eg_o (lb-in).

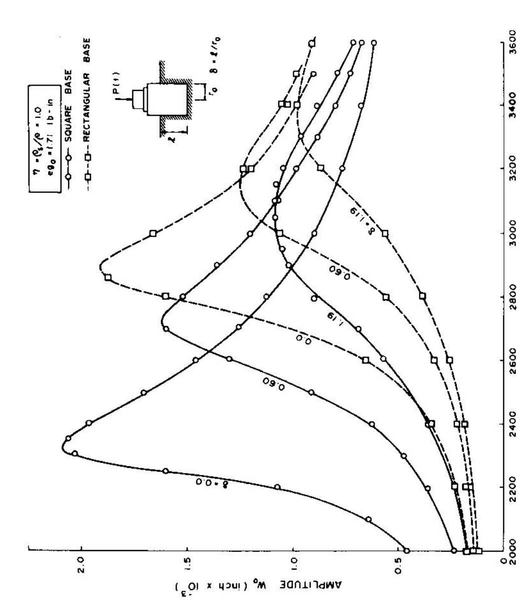

FIGURE 7 VERTICAL VIBRATIONS OF SQUARE AND RECTANGULAR FOOTINGS EMBEDDED IN UNDISTURBED SOIL. (EQUAL BASE AREAS MASSES AND EXCITATION)

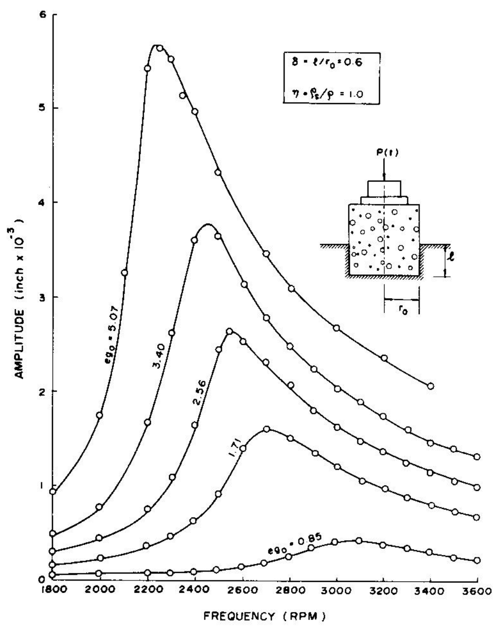

FIGURE 6 VERTICAL VIBRATION OF SQUARE FOOTING EMBEDDED IN UNDISTURBED SOIL (RELATIVE EMBEDMENT l/r_o = 0.6, VARIOUS ECCENTRIC MOMENTS eg_o)

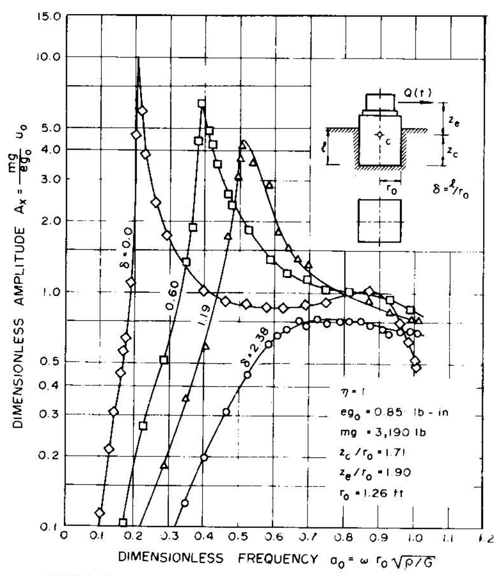

FIGURE 8 HORIZONTAL VIBRATIONS OF CENTRE OF GRAVITY OF SQUARE FOOTINGS EMBEDDED IN UNDISTURBED SOIL (SAME ECCENTRIC MOMENT, VARIOUS EMBEDMENTS)

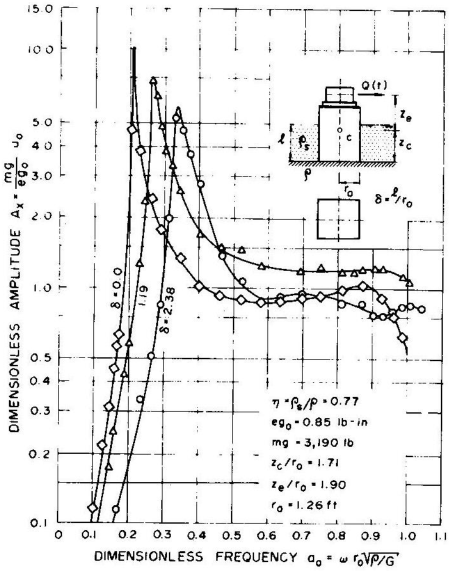

FIGURE 9 HORIZONTAL VIBRATIONS OF CENTRE OF GRAVITY OF SQUARE FOOTING EMBEDDED IN BACK FILL (SAME ECCENTRIC MOMENT, VARIOUS EMBEDMENTS)

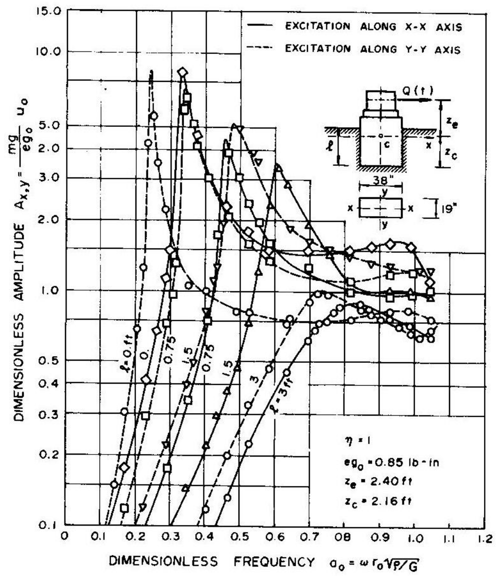

FIGURE 10 HORIZONTAL VIBRATIONS OF CENTRE OF GRAVITY OF RECTANGULAR FOOTING EMBEDDED IN UNDISTURBED SOIL (SAME ECCENTRIC MOMENT, VARIOUS EMBEDMENTS, $I_x = 181.5$ lb-ft-sec^2, $I_y = 239.6$ lb-ft-sec^2)

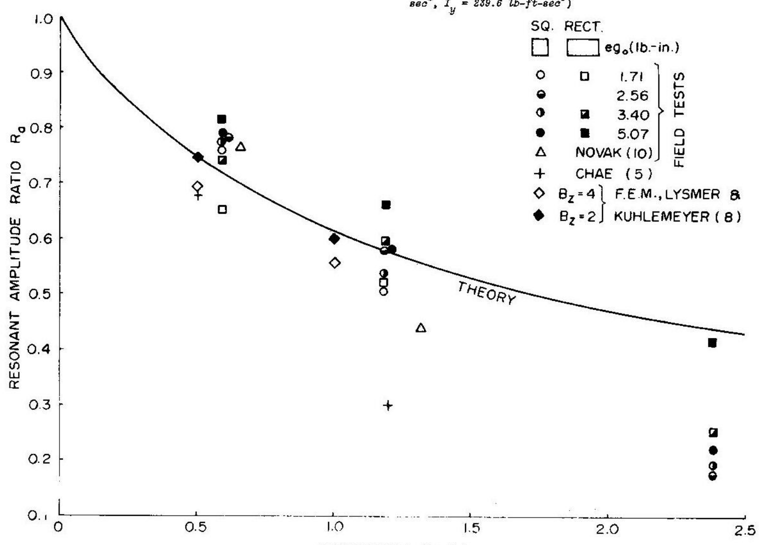

FIGURE 11 COMPARISON OF THEORETICAL AND EXPERIMENTAL RESONANT AMPLITUDE RATIOS FOR VERTICAL VIBRATIONS OF FOOTINGS EMBEDDED IN UNDISTURBED SOIL

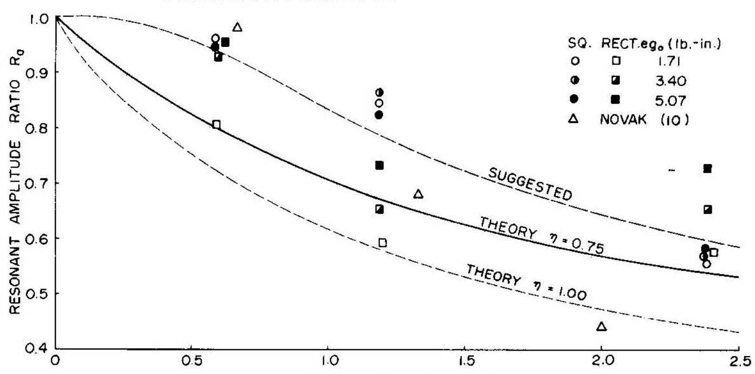

FIGURE 12 COMPARISON OF THEORETICAL RESONANT AMPLITUDE REDUCTION WITH FIELD TESTS OF FOOTINGS EMBEDDED IN BACK FILL HAVING DENSITY RATIO $\eta = \rho_s/\rho = 0.77$

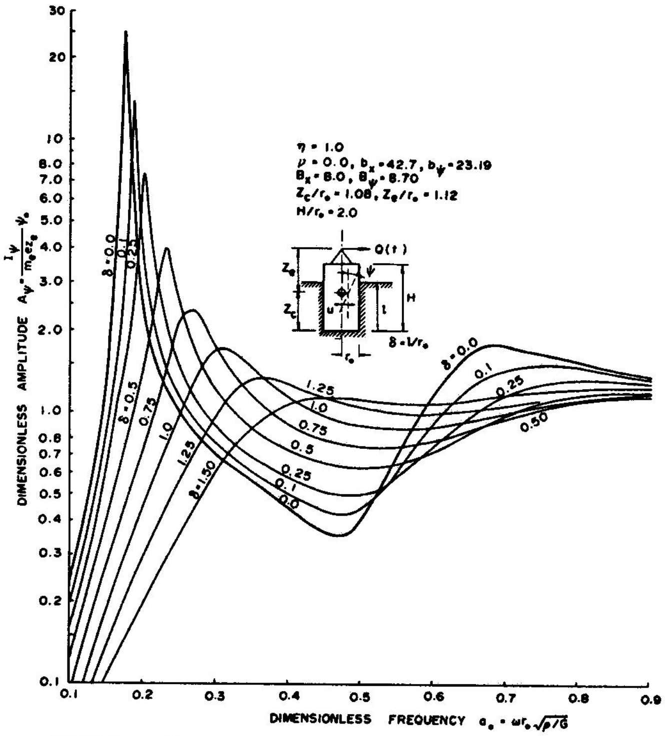

FIGURE 13 EXAMPLE OF THEORETICAL RESPONSE IN COUPLED HORIZONTAL TRANSLATION AND ROCKING (ROCKING COMPONENT, UNDISTURBED SOIL)

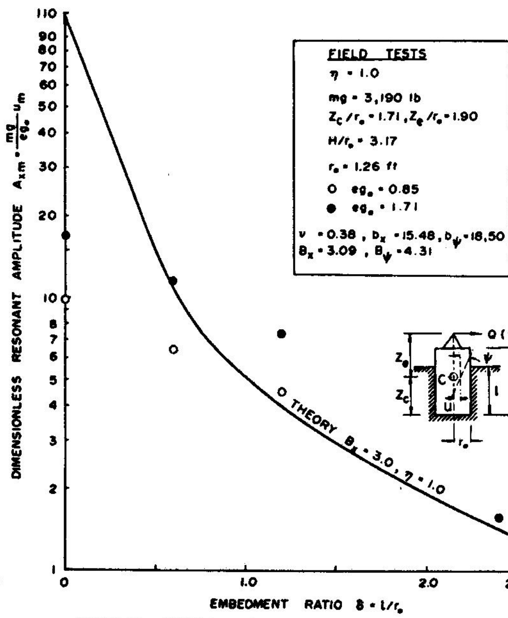

FIGURE 15 COMPARISON OF THEORETICAL AND EXPERIMENTAL RESONANT AMPLITUDES (FIRST RESONANCE, UNDISTURBED SOIL)

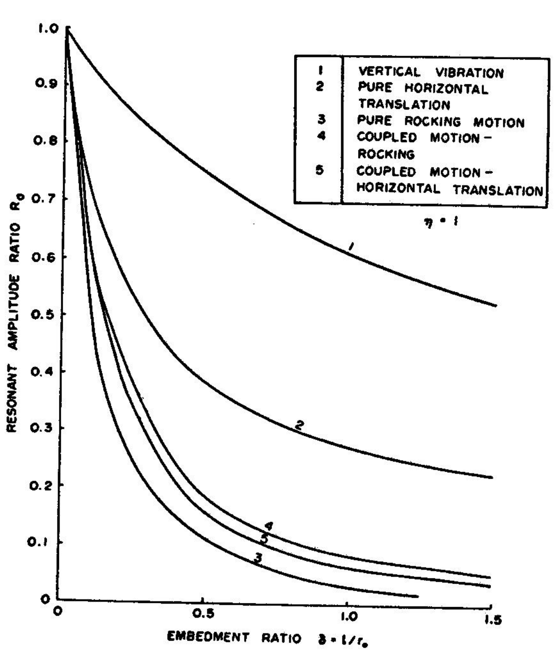

FIGURE 14 THEORETICAL RESONANT AMPLITUDE REDUCTION FOR VARIOUS VIBRATION MODES (UNDISTURBED SOIL)

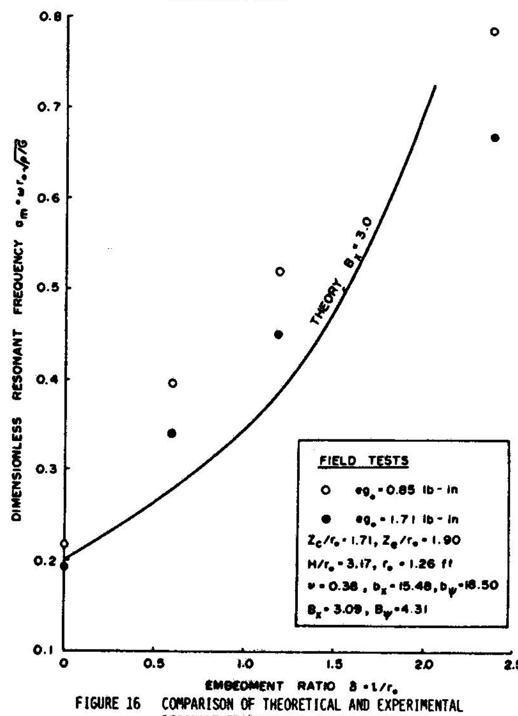

FIGURE 16 COMPARISON OF THEORETICAL AND EXPERIMENTAL RESONANT FREQUENCIES (FIRST RESONANCE, UNDISTURBED SOIL)

DISCUSSION OF PAPER NO. 7

THE EFFECT OF EMBEDMENT ON FOOTING VIBRATIONS

by

M. Novak and Y. O. Beredugo

Question by: K.G. Asmis

How would you adopt your theoretical analysis to obtain a nonlinear analysis? Also, what mathematical form of nonlinearity would you use to model the nonlinear behaviour?

Reply by: M. Novak

To introduce a true nonlinearity into the half-space theory would be very difficult. Fortunately, it does not seem necessary with steady-state vibrations considered in the paper. The nonlinear features of response curves can be described by a linear theory by introducing a shear modulus (or an equivalent spring constant) which is dependent upon the level of the dynamic stress. In other words, it seems possible to assume that the soil linearizes after many loading cycles within the range of the vibration amplitudes ($-A,+A$). If the amplitude is changed the soil linearizes again, however it exhibits a different stiffness. Such properties were called nongenuine (or pseudo-) nonlinearity as the vibration at any steady amplitude is linear while the response curve features typical nonlinearities (Refs. 9, 10).

The inverse problem is to derive data from a nonlinear response curve obtained experimentally. This is discussed in Ref. 11.

Question by: W.D.L. Finn

How do you assess lateral resistance in the case of the back fill footing. You took one for your embedded footing and reduced it down as low as 0.75. I am not certain how you assessed these factors.

Reply by: M. Novak

One of the advantages of this approximate analysis approach is that we are in a position to express the equivalent stiffness and equivalent damping using some formula for their computation. We get these equivalents as a function of frequency, shear modulus and density. Choosing, say density and shear modulus of side fill, we can get various stiffnesses and equivalent dampings. The additional bonus we get is that having these qualities, we can introduce the reactions of embedded footings into the calculations of more complicated structures; for example, a tall building which sits on some embedded caisson, can be solved and we can introduce reactions as in any other kind of lumped parameter model with the only change being that our stiffness and damping are frequency-dependent but it is not necessary to take any additional care about the half-space or about the soil.